Failure probability of river dikes strengthened with structural elements

Chances d'échec des digues rivières renforcées par les éléments structuraux

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ABSTRACT

Nowadays in the Netherlands river dikes sometimes are strengthened with structural elements like sheet pile walls etc. At present a project is carried out to determine inundation risks of so-called dike rings. This requires calculation of the failure probabilities of the dikes from which these dike rings consists. In this paper a method is presented to calculate on a comparatively quick and easy way an estimate of the failure probability of a river dike strengthened with structural elements. The method makes use of the PLAXIS finite element model, particularly the so-called Phi/C reduction option for the calculation of safety factors. In this paper theoretical background and a practical example are described.

RÉSUMÉ

De nos jours aux Pays-Bas, les digues fluviales sont parfois fortifiées par les éléments structuraux telles que les cloisons de palplanches, etc. Un projet est exécuté actuellement pour évaluer le risque de l'inondation des soi-disant cercles des digues. Cela exige le calcul des risques d'échec des digues qui composent les cercles. Dans cet article, une méthode est présentée pour calculer, d'une façon relativement rapide et facile, une estimation des. La méthode profite du modèle de simulation numérique des éléments finis PLAXIS, particulièrement de soi-disant l'option de réduction Phi/C pour le calcul de facteurs de sécurité. Dans cet article, le contexte ainsi q'un exemple pratique sont présentés.

1 INTRODUCTION

There is a general consensus among Netherlands dike engineers that the Finite Element Method (FEM) is the method of choice for the calculation of river dikes strengthened with structural elements. Calculation of failure probabilities of these dikes with existing level II probabilistic models is a time consuming and complex task, because of the many uncertainties that must be considered. Besides it needs FEM calculations with shear strength parameters near soil failure. FEM programs often not perform very well with shear strength parameters near failure.

To avoid these difficulties, in this paper a method is presented to calculate an estimates of the failure probability of a river dike strengthened with structural elements. The proposed method requires only a small number of FEM calculations with Phi/C reductions. From the safety factors following from these calculations, the number of independent soil layers in the failure mechanism and the statistical distributions of shear strength parameters, wall friction, ground water levels, geometry, external loads, high-water level and strength of structural elements, an estimate of the failure probability can be calculated on a comparatively quick and easy way.

2 STARTING POINTS

Although parts of the proposed method are based on Level III probabilistic principles, also linearisations are used to come to a manageable system of equations. For this reason in general the method must be considered as a Level II probabilistic method.

PLAXIS implemented Phi/C reduction option

The method makes use of the PLAXIS Finite element model implemented Phi/C reduction method as described by Brinkgreve and Bakker (1990). During Phi/C reduction the shear strength of the soil is reduced, until a failure mechanism arises and no further reduction is possible. The proportion between the initial shear strength and the shear strength at failure is the so-called Multiplier of Safety Factor (Σ-MSF), defined as:

$$\Sigma - MSF_c = \frac{C_a + \sigma^{'}.\tan\varphi_a}{C_c + \sigma^{'}.\tan\varphi_c} \qquad \textbf{(2.1)}$$

Where C_a and φ_a are the initial drained shear strength parameters, C_c and φ_c the shear strength parameters at failure and Σ-MSF_c the safety factor at failure. Further on in this paper Σ-MSF is indicated as Fv.

Coulomb friction criterion and undrained shear strength

A basic assumption is that geotechnical failure can be described with a Coulomb friction criterion:

$$\tau = C + \sigma_{\perp}.\tan\varphi \qquad \textbf{(2.2)}$$

Where τ is the shear strength and $\sigma_{\perp}$ the normal stress perpendicular to the failure plane. Failure of river dikes usually occurs under high-water conditions. When the dike is build up from clay and loam, this usually is an undrained process.
In this paper it is assumed that failure can be described by an undrained shear strength Cu. Supposing that the water in the soil may be considered as incompressible in proportion to the stiffness of the soil skeleton, it can be shown that an equivalent undrained shear strength Cu can be calculated from:

$$Cu(C,\varphi) = \sigma^{'}_{p}.\sin\varphi + C.\cos\varphi \qquad \textbf{(2.3)}$$

Where σ'_p is the mean effective stress obtained from $\sigma'_p = 0{,}5.\ \sigma'_v.(1+K_0)$ with K_0 the coefficient of earth pressure at rest.
For Cu calculated from equation (2.3) it can be shown that the following relation exists between the safety factor Fv_c at failure, the undrained shear strength Cu_c at failure, and Cu_a calculated from C_a and φ_a:

$$\frac{Cu_a}{Cu_c} = \sqrt{\frac{Fv_c^2 + \tan^2\varphi_a}{1 + \tan^2\varphi_a}} \qquad \textbf{(2.4)}$$

Assessment of shear strengths in under ground model

Soil properties, as determined from small test samples within a layer, usually reveal considerable spatial variability within the

soil unit, covered by the test sample set. In a geotechnical (failure) analysis a soil parameter often reflects in a way the average of a soil property related to the volume or surface affected by the failure mechanism. Therefore in the assessment of soil parameters for geotechnical analyses due account should be taken for both spatial variability (as found from test samples), as well as averaging of spatial fluctuations within the volumes or surfaces affected by the failure mechanisms (JCSS 2002).
The JCSS Probabilistic Model Code advocates therefore the use of Homogeneous Random Field Models to describe the non-systematic spatial variability pattern of soil properties.
Such models enable assessment of soil parameters for a geotechnical analysis in a more objective way. Key feature of such a model is the spatial correlation model, i.e. the autocorrelation function. The autocorrelation function type, applied in coherence with regional test data, in the Dutch code for design of river dikes, reads:

$$\rho(\Delta x, \Delta z) = \exp - \left(\left(\frac{\Delta x}{D_h} \right)^2 \right) \cdot \left((1-a) + a.\exp\left(- \left(\frac{\Delta z}{D_v} \right)^2 \right) \right) \quad \textbf{(2.5)}$$

where □(□x,□z) is the correlation between shear strengths in any two points within one soil layer, separated with distances □x and □y in horizontal and vertical direction. D_h and D_v are auto correlation parameters (D_h is in the order of tens of meters, whereas D_v is in the order of some decimetres) and (a) is the ratio between the local point variation (vertical direction) and the total regional variation (variation of the mean value of the local shear strengths in horizontal direction). In practical cases carried out in the Netherlands, (a) is estimated to be 0,75 to 1,0.

At present the above-mentioned procedure is not available in Finite Element Models, so for the analysis of dikes strengthened with structural elements another method must be used.
In the proposed method in this paper it will be assumed that fluctuations of shear strength in a vertical direction will be completely averaged within the volume affected by the analysis, whereas averaging of fluctuations in a horizontal direction will be ignored. The standard deviation of the shear strength of a soil unit or layer, taking statistical uncertainty of the expected mean value into account, may then be calculated as:

$$\sigma_{\bar{c}} = \sigma_{\hat{c}} \cdot \sqrt{(1-a) + \frac{1}{n}} \quad \textbf{(2.6)}$$

Where $□_{\bar{c}}$ is standard deviation of the shear strength of a layer, $□_{□}$ standard deviation from shear tests, (n) number of shear tests and (a) ratio between the local point variation and the total regional variation as described before.
In the proposed method it is assumed that there is no correlation between the soil properties of different soil layers.

Uncertainties in high-water level
Uncertainties in high-water level can be described with a Gumbel distribution function:

$$F_1(H) = \exp\left(- \exp\left(- \frac{2{,}3(H-u)}{B} \right) \right) \quad \textbf{(2.7)}$$

Where H is water level, $F_1(H)$ yearly probability that H shall not be exceeded, B is decimation height and (u) determined from:

$$u = MHW + B. \frac{\ln\left(\ln\left(\frac{1}{1-P_{MHW}} \right) \right)}{\ln(10)} \quad \textbf{(2.8)}$$

Where MHW is Normative High Water and P_{MHW} is yearly probability of occurrence of MHW.

Other uncertainties
In addition to the uncertainties in shear strength and high-water level, also uncertainties in wall friction, ground water levels, geometry, external loads and strength of structural elements are considered. It is assumed that these uncertainties may be described with normal (Gausian) distribution functions i.e. with mean values (□) and standard deviations (□).

3 THEORY

The request was a method that with a minimum of information and only information what can be acquired easy and is understandable for practical engineers, results in an estimate of the failure probability of a river dike strengthened with structural elements. In this chapter the theory of the method is described.

Stability factor
The stability factor for soil failure can be written as a linear combination of the undrained shear strengths from the soil layers from which the failure mechanism consists:

$$Fs = \frac{\sum_{i=1}^{n} w_i.Cu_i}{\sum_{i=1}^{n} w_i.Cu_{c,i}} \quad \textbf{(3.1)}$$

Where Fs is stability factor, (i) number of independent soil layers in the failure mechanism, Cu_i undrained shear strength, $Cu_{c,i}$ undrained shear strength at failure and w_i weight factors depending on size and incremental strains of the particular soil layer. In the continuation of this paper it is supposed that the weight factors satisfy □ $w_i = 1$.

Linearising
Cu calculated from drained shear strength parameters C and □, is a non-linear function of C and □ and consequently also Fs.
In that case Fs must be linearised in a design point S*.
After linearisation Fs no longer by definition satisfies that for $Cu_i = 0$, $Fs = 0$. In linearised form Fs can be written as:

$$Fs = 1 + K.\sum_{i=1}^{n} w_i.(Cu_i - Cu_{c,i}) \quad \textbf{(3.2)}$$

With:

$$K \approx \left(\sum_{i=1}^{n} w_i.Cu_{c,i} \right)^{-1} \quad \textbf{(3.3)}$$

Note that for □ is = the equations (3.1) and (3.2) are identical.

Soil failure function
The soil failure function Z_g is defined as:

$$Z_g = Fs - 1 \quad \textbf{(3.4)}$$

From equation (3.2) it follows:

$$Z_g = K.\sum_{i=1}^{n} w_i.(Cu_i - Cu_{c,i}) \quad \textbf{(3.5)}$$

From equation (3.5) it follows for the mean value □(Z_g) and standard deviation □(Z_g):

$$\mu(Z_g) = K.\sum_{i=1}^{n} w_i.(\mu(Cu_i) - Cu_{c,i}) \quad \textbf{(3.6)}$$

and:

$$\sigma(Z_g) = K.\sqrt{\sum_{i=1}^{n} w_i^2.\sigma^2(Cu_i)} \quad \textbf{(3.7)}$$

Reliability index for soil failure
From □(Z_g) and □(Z_g) follows the reliability index for soil failure:

$$\beta^{\circ}_{g|MHW} = \frac{\mu(Z_g)}{\sigma(Z_g)} = \frac{\sum_{i=1}^{n} w_i.(\mu(Cu_i) - Cu_{c,i})}{\sqrt{\sum_{i=1}^{n} w_i^2.\sigma^2(Cu_i)}} \quad \textbf{(3.8)}$$

Where the superscript ° indicates that $\beta^{\circ}_{g\,|MHW}$ is calculated from fixed values for all parameters, except C and φ. From $\beta^{\circ}_{g\,|MHW}$, the failure probability $Pf^{\circ}_{g\,|MHW}$ can be calculated:

$$Pf^{\circ}_{g|MHW} = \Phi(-\beta^{\circ}_{g|MHW}) \qquad \textbf{(3.9)}$$

With (Φ) the normal distribution function.

Reliability index for a single layer failure mechanism

For a single layer failure mechanism, the number of soil layers (i) and weight factors w_i are equal to 1,0. Then the reliability index for soil failure becomes:

$$\beta^{\circ}_{g|MHW} = \frac{\mu(Z_g)}{\sigma(Z_g)} = \frac{\mu(Fs)-1}{\sigma(Fs)} = \frac{\frac{\mu(Cu)}{Cu_c}-1}{\frac{\sigma(Cu)}{Cu_c}} \qquad \textbf{(3.10)}$$

Reliability index for a multi layer failure mechanism

For a multi layer failure mechanism, $\beta^{\circ}_{g|MHW}$ can be determined only when the weight factors w_i are known. Determination of w_i is a complex task, because they depend on the incremental strains in the failure mechanism. When w_i are unknown, they must be considered as uncertain parameters. All partitions of w_i that satisfy $\Sigma\, w_i = 1{,}0$ have a possibility to be the right partition. With equations (3.8) and (3.9) for every partition of w_i a reliability index $\beta^{\circ}_{g\,,j|MHW}$ and a failure probability $Pf^{\circ}_{g\,,j|MHW}$ can be determined. The best estimate for $Pf^{\circ}_{g|MHW}$ is the average of the failure probabilities following from all possible partitions of w_i.

Monte-Carlo simulation

To determine $Pf^{\circ}_{g\,|MHW}$, a Monte-Carlo simulation is used to perform random drawings of w_i . Each drawing is carried out by a random cut up of an interval (0..1) in so many parts as there are soil layers in the failure mechanism. With equations (3.8) and (3.9) each drawing results in a reliability index $\beta^{\circ}_{g\,,j|MHW}$ and a failure probability $Pf^{\circ}_{g\,,j|MHW}$. Then the best estimate for $\beta^{\circ}_{g\,|MHW}$ follows from:

$$\beta^{\circ}_{g|MHW} = -\Phi^{-1}(Pf^{\circ}_{g|MHW}) = -\Phi^{-1}\left(\frac{1}{m}\sum_{j=1}^{m}\Phi(-\beta^{\circ}_{g,j|MHW})\right) \qquad \textbf{(3.11)}$$

Where Φ^{-1} is the inverse normal distribution function, (m) the number of drawings and (j) a particular drawing. To assure a reliable determination of $\beta^{\circ}_{g\,|MHW}$, a sufficient number of drawings must be carried out. Beside a best estimate for $\beta^{\circ}_{g\,|MHW}$, also an interval can be determined in which the reliability index can vary with a certain likelihood, when the weight factors should been determined exactly. So, statements as: "With a likelihood of 80 %, this construction shall have a reliability index between 3,7 and 4,3" are also possible.

Averaging of uncertainties

Equation (3.8) also shows the so-called averaging of uncertainties. In this equation the denominator contains factors w_i^2.
Because for a single layer mechanism w_1 is 1.0 , $\Sigma(w_i^2) = 1{,}0$. But for a multi layer mechanism, $\Sigma(w_i^2)$ only can be 1,0 in the unlikely event that one weight factor is 1,0 and all others are 0. In all other cases the weight factors w_i are smaller then 1,0 and consequently $\Sigma(w_i^2) < 1{,}0$. So, for the same Safety Factor Fv, uncertainties in soil strength and no correlation between layers, a multi layer mechanism shall result in a higher reliability index then a single layer mechanism, see figure 3.1.

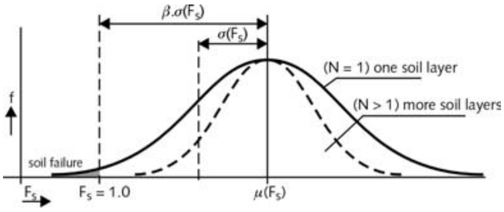

Figure 3.1 Averaging of uncertainties

Other uncertainties

It can be shown that the influence on the reliability index from uncertainties in groundwater levels, geometry, wall friction, external loads and other uncertainties can be taken into account with the following equation:

$$\beta_{g|MHW} = \frac{\beta^{\circ}_{g\,|MHW}}{\sqrt{\left(1+\sum_{i=1}^{n}\left(\frac{\partial\beta^{\circ}_{g|MHW}}{\partial x_i}\right)^2.\sigma^2(x_i)\right)}} \qquad \textbf{(3.12)}$$

Where $\beta_{g|MHW}$ is the reliability index for soil failure including uncertainties, x_i uncertain parameter (i), $\sigma(x_i)$ standard deviation of x_i and (n) number of uncertain parameters. In this equation the differentials $d\beta^{\circ}_{g\,|MHW}/dx_i$ follows from:

$$\frac{\partial\beta^{\circ}_{g|MHW}}{\partial x_i} = \frac{\partial\beta^{\circ}_{g|MHW}}{\partial Fv_c}.\frac{\partial Fv_c}{\partial x_i} \qquad \textbf{(3.13)}$$

Where:

$$\frac{\partial\beta^{\circ}_{g|MHW}}{\partial Fv_c} = \frac{\partial\beta^{\circ}_{g|MHW}}{\partial\mu(Fs)}.\frac{\partial\mu(Fs)}{\partial Fv_c} \qquad \textbf{(3.14)}$$

From equations (2.4) and (3.10) it can be shown this is equal to:

$$\frac{\partial\beta^{\circ}_{g|MHW}}{\partial Fv_c} = \frac{Fv_c}{\sigma(Fs).\left(Fv_c^{\,2}+\tan^2(\overline{\varphi})\right)} \qquad \textbf{(3.15)}$$

Where $\overline{\varphi}$ is the friction angle that for a single layer failure mechanism should result in the average stability factor from the Monte-Carlo simulation. The derivates dFv_c/dx_i in equation 3.13 can be derived from PLAXIS calculations with $x_i+\Delta x_i$ while all other parameters remain unchanged. From equation 3.12 it follows for the probability of soil, including the above mentioned uncertainties:

$$Pf_{g|MHW} = \Phi(-(\beta_{g|MHW})) \qquad \textbf{(3.16)}$$

Failure probability of structural elements

Figure 3.2 shows the failure functions for both soil failure, $Z_g = 0$, and sheet pile failure, $Z_d = 0$.

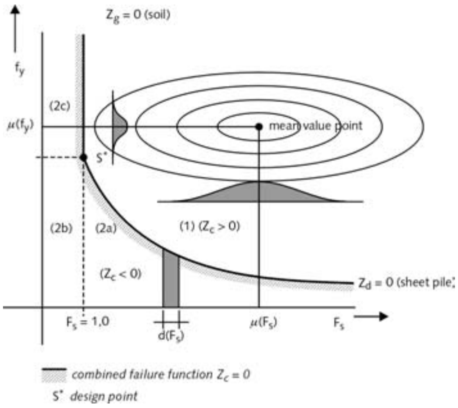

Figure 3.2 Failure functions for soil and sheet pile

In figure 3.2 is f_y the (uncertain) sheet pile strength, while the appearing stress in the sheet pile $f_{y,d}(Fs)$ follows from $Z_d = (f_y - f_{y,d}(Fs)) = 0$. The domains (2b) and (2c), i.e Fs $\leq$ 1,0 represents soil failure. The domain (2a), i.e. Fs $\geq$ 1,0 and f_y below Z_d represents sheet pile failure and the domain (1), f_y above Z_d and Fs > 1,0 , represent no failure et all. $f_{y,d}(Fs)$ can be determined from a Phi/C reduction or from a number of calculations with immediately reduced shear strengths parameters. For fixed values of Fs $\geq$ 1 , the likelihood of sheet pile failure follows from:

$$Pf_{d|MHW}\,|\,Fs = \Phi(-\beta_{d|MHW}\,|\,Fs) = \Phi\left(-\frac{\mu(f_y)-f_{y,d}(Fs)}{\sigma(f_y)}\right) \qquad \textbf{(3.17)}$$

In the case of an anchor, a comparable expression exists for anchor failure. Also expressions can be derived for the likelihood of sheet pile and anchor failure, sheet pile and no anchor failure, anchor and no sheet pile failure, and no failure et all. From equation 3,17 it follows for the probability of sheet pile failure for a fixed value of MHW and all values of Fs □ 1 :

$$Pf_{d|MHW} \mid Fs \geq 1 = \int_{F_s=1}^{\infty} f(Fs).(Pf_{d|MHW} \mid Fs).dFs \qquad \textbf{(3.18)}$$

Where f(Fs) is the derivate to Fs of the distribution function of Fs. For Fs < 1 the failure probability of the sheet pile is undetermined, because for Fs < 1 , $f_{y,d}(Fs)$ can not be determined.

Failure probability of the construction

From figure (3.2) and equations (3.16) and (3.18) it follows for the failure probability for the whole construction $Pf_{c|MHW}$, given a fixed value of MHW:

$$Pf_{c|MHW} = Pf_{g|MHW} + Pf_{d|MHW} \mid Fs \geq 1 \qquad \textbf{(3.19)}$$

Uncertainties in high-water level

Uncertainties in the high-water level can be taken into account using the following equation:

$$Pf_c = \int_{H=-\infty}^{H=\infty} f_1(H).Pf_{c|H}.dH \qquad \textbf{(3.20)}$$

Where Pf_c is failure probability including uncertainties in high-water level, $f_1(H)$ is the derivate to H of the Gumbel distribution of the high-water level, see equation (2.7), and $Pf_{c|H}$ determined from $Pf_{c|H}$ = □(-□$_{c|H}$). The determination of $Pf_{c|H}$ requires the determination of □$_{c|H}$ for all possible water levels H. This is an extensive task. However, it may be assumed that water levels near Normative High Water provides larger contributions to Pf_c then lower water levels. So it is assumed that □$_{c|H}$ may be linearised in a design point, for what is chosen the Normative High Water MHW, resulting in the following linearised equation for □$_{c|H}$:

$$\beta_{c|H} = \beta_{c|MHW} + \frac{\Delta\beta_{c|H}}{\Delta H}.(H - MHW) \qquad \textbf{(3.21)}$$

Determination of □□$_{c|H}$/□H requires determination of the reliability index □$^{\circ}_{g\ |MHW+□H}$ for a fixed water level MHW+□H. This can be calculated from:

$$\beta^{\circ}_{g|MHW+\Delta H} = \beta^{\circ}_{g|MHW} + \frac{\partial\beta^{\circ}_{g|MHW}}{\partial Fv_c}.(Fv_{c|MHW+\Delta H} - Fv_{c|MHW}) \qquad \textbf{(3.22)}$$

Where $Fv_{c|MHW+□H}$ can be calculated from a PLAXIS Phi/C reduction with water level (MHW+□H) while all other parameters remain unchanged. After □$^{\circ}_{g|MHW+□H}$ is determined, the influence of other uncertainties and structural elements can be calculated as discussed before, resulting in a reliability index □$_{c|MHW+□H.}$ Then □□$_{c|H}$/□H follows from:

$$\frac{\Delta\beta_{c|H}}{\Delta H} = \frac{\beta_{c|MHW+\Delta H} - \beta_{c|MHW}}{\Delta H} \qquad \textbf{(3.23)}$$

4 RIVER DIKE STRENGTHENED WITH SHEETPILE

As an example of the proposed method, the results of a calculation of the failure probability of a river dike strengthened with a not anchored sheet pile are presented; see Figure 4.1 and Table 1. In Table 1, NAP means: “Meter above Normal Amsterdam Water Level”. From a PLAXIS calculation with Phi/C reduction for mean values and Normative High Water, a Safety Factor Fv_c = 1,28 was calculated and a failure mechanism including 5 independent soil layers. Given a fixed value of MHW = NAP + 10,99, from the uncertainties in soil strength exclusive other uncertainties, a reliability index of 3,34 was calculated. Besides it was found that a more elaborated calculation, including a precise determination of the weight factors, with a likelihood of 80% should result in a reliability index between 3,07 and 4,21.

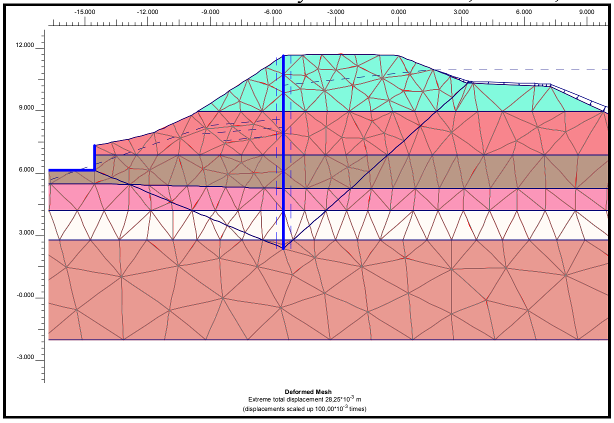

Figure 4.1. River dike strengthened with sheet pile

The reliability index inclusive other uncertainties, is calculated at 2,99. When this should be the final result, this should lead to a disapproval of the construction. However, after taking into account the Gumbel distribution of the high-water level, the reliability index increases until 4,08, corresponding with a yearly failure probability of 2,25 10^{-5}.

Table 1: List of parameters

Soil layers (top to bottom)		**□(C)**	**□(Phi)**	**□(C)**	**□(Phi)**
Dike soil (1)		4,53	26,38	0,95	3,03
Dike soil (2)		4,53	26,38	0,95	3,03
Dike soil (3)		4,53	26,38	0,95	3,03
Clayey sand		5,11	26,73	1,07	3,07
Silty sand		5,13	23,89	1,08	2,75
Normative High Water [NAP]		**yearly probability [1/n]**		**decimation height [m]**	
10,99		0,002		0,91	
Other uncertainties		**(□)**		**(□)**	
Sheet pile toe level	[NAP]	2,38		0,10	
Phreatic line left	[NAP]	8,61		0,70	
Phreatic line right	[NAP]	10,24		0,70	
Wall friction	[%]	100		5	
Sheet pile strength	[mPa]	360,00		18,00	

5 SUMMARY AND CONCLUSIONS

With the proposed method it is possibility to calculate an estimate of the reliability index of a dike strengthened with structural elements on a comparatively quick and easy way.

The advantage of the method is the reduction in the number of FEM calculations and that it only needs information that can be acquired easily and is understandable for practical geotechnical engineers. The method makes use of the PLAXIS Finite Element Program, especially the Phi/C reduction option. The method is implemented in a number of spreadsheets.

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